MA 111: Precalculus, NCSU

Module 2 In-Class Problems: Absolute Value Function

Section 1.6, #62:
Students who score within 18 points of the number 82 will pass a particular test. Write this statement using absolute value notation and use the variable x for the score.

Section 1.6, #64:
The tolerance for a ball bearing is 0.01. If the true diameter of the bearing is to be 2.0 inches and the measured value of the diameter is x inches, express the tolerance using absolute value notation.

Section 1.6, not in textbook

Solve the following absolute value inequalities. Show your answers on a number line and in standard interval notation for your final answers.

a. $|2x - 4| \geq 8$

b. $|2x - 4| < 8$

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Module 2 In-Class Problems: Absolute Value Function

Section 1.6, #62:
Students who score within 18 points of the number 82 will pass a particular test. Write this statement using absolute value notation and use the variable x for the score.

A student passes the test if their score is within 18 below or 18 above 82. Then if we subtract the score x from 82, we should have $-18 \leq 82 - x \leq 18$ (the difference is negative if the student scores higher than 82). In absolute value form, this means

$$|82 - x| \leq 18$$

Section 1.6, #64:
The tolerance for a ball bearing is 0.01. If the true diameter of the bearing is to be 2.0 inches and the measured value of the diameter is x inches, express the tolerance using absolute value notation.

Similar to the previous question, we can say that the difference between the exact/true diameter and the measured diameter x must fall between -0.01 and 0.01. Then we have

$$|2.0 - x| \leq 0.01$$

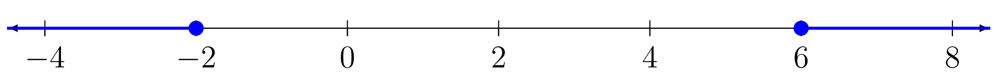
Section 1.6, not in textbook
Solve the following absolute value inequalities. Show your answers on a number line and in standard interval notation for your final answers.

a. $|2x - 4| \geq 8$

Since we want to know when the absolute value of $2x - 4$ is greater than or equal to 8, we will consider two cases:

$$\begin{aligned} 2x - 4 &\geq 8 \quad \text{and} \quad 2x - 4 \leq -8 \\ 2x &\geq 12 \quad \text{and} \quad 2x \leq -4 \\ x &\geq 6 \quad \text{and} \quad x \leq -2 \end{aligned}$$

The solution in standard interval notation is $(-\infty, -2] \cup [6, \infty)$. On a number line, we have:

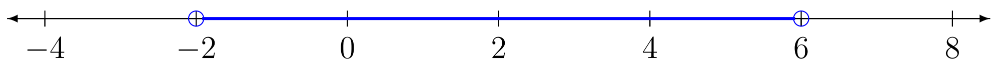


b. $|2x - 4| < 8$

Here we want to know when the absolute value of $2x - 4$ is less than 8:

$$-8 < 2x - 4 < 8$$
$$-4 < 2x < 12$$
$$-2 < x < 6$$

The solution in standard interval notation is $(-2, 6)$. On a number line, we have:



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Module 6 In-Class Problems: Quadratic Functions

Section 3.2, #86:
Find the dimensions of the rectangular corral split into 2 pens of the same size producing the greatest possible enclosed area given 300 feet of fencing.

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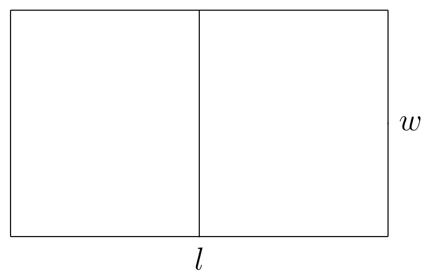
Module 6 In-Class Problems: Quadratic Functions

Section 3.2, #86:
Find the dimensions of the rectangular corral split into 2 pens of the same size producing the greatest possible enclosed area given 300 feet of fencing.

First, we must draw a picture to understand what we are calculating:



We know that all the sides must add up to the fencing available - 300 ft. Then we have:

$$2l + 3w = 300$$

We'll solve this equation for one of the variables. Here, we will solve for w (at the end of this problem, we will solve for l as well):

$$3w = 300 - 2l$$
$$w = \frac{300}{3} - \frac{2l}{3}$$
$$= 100 - \frac{2}{3}l$$

What we really want is to find the maximum area possible in this situation, so let's find the area function for these pens:

$$A = l \cdot w$$
$$A(l) = l\left(100 - \frac{2}{3}l\right)$$
$$= 100l - \frac{2}{3}l^2$$

Although it's not in the order we're used to, this is a quadratic function in l. Since the coefficient of l^2 is negative, this is a concave down parabola. The highest point on the graph will give the maximum value of the area function. This occurs at the vertex.

Let's find the l-coordinate of the vertex:

$$\begin{aligned} l &= -\frac{b}{2a} \\ &= -\frac{100}{2\left(-\frac{2}{3}\right)} \\ &= -\frac{100}{-\frac{4}{3}} \\ &= -100\left(-\frac{3}{4}\right) \\ &= 75 \end{aligned}$$

Then the maximum possible area is given by $A(75)$:

$$\begin{aligned} A(75) &= 100(75) - \frac{2}{3}(75)^2 \\ &= 7500 - \frac{2}{3}(5625) \\ &= 3750 \end{aligned}$$

The maximum possible area is 3750 ft^2.

Finally, let's find the corresponding width:

$$\begin{aligned} w &= 100 - \frac{2}{3}(75) \\ &= 100 - 50 \\ &= 50 \end{aligned}$$

The dimensions of the corral that maximize the area are 75 ft long and 50 ft wide.

Solving for l instead:

$$\begin{aligned} 2l &= 300 - 3w \\ l &= \frac{300}{2} - \frac{3w}{2} \\ &= 150 - \frac{3}{2}w \\ A(w) &= \left(150 - \frac{3}{2}w\right)w \\ &= 150w - \frac{3}{2}w^2 \\ w \text{ (vertex)} &= -\frac{150}{2\left(-\frac{3}{2}\right)} \\ &= 50 \\ l &= 75 \end{aligned}$$

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Module 7 In-Class Problems: Higher Degree Polynomials

Section 3.3, #66:
An oil slick is expanding as a circle. The radius of the circle is increasing at the rate of 20 meters per day. Express the area of the circle as a function of d, the number of days elapsed.

Section 3.3, #69:
An open box is to be constructed by cutting out square corners of x-inch sides from a piece of cardboard 8 inches by 8 inches and then folding up the sides. Express the volume of the box as a function of x.

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Module 7 In-Class Problems: Higher Degree Polynomials

Section 3.3, #66:
An oil slick is expanding as a circle. The radius of the circle is increasing at the rate of 20 meters per day. Express the area of the circle as a function of d, the number of days elapsed.

First, let's find a function for the radius with respect to number of days elapsed d. The radius of the circle is expanding 20 m every day. Then:

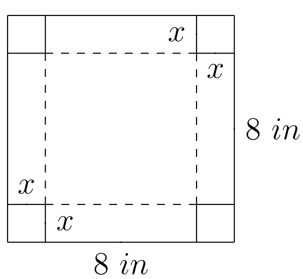
$$r(d) = 20d$$

Now, we know that the area of a circle is $A(r) = \pi r^2$. Using the function we found for the radius, we'll find $A(d)$:

$$\begin{aligned} A(d) &= \pi\,(20d)^2 \\ &= \pi\left(400d^2\right) \\ &= 400\pi\, d^2 \end{aligned}$$

The area of the oil slick with respect to d is $A(d) = 400\pi\, d^2$ in m^2.

Section 3.3, #69:
An open box is to be constructed by cutting out square corners of x-inch sides from a piece of cardboard 8 inches by 8 inches and then folding up the sides. Express the volume of the box as a function of x.



To form the box, we would fold along the dotted lines. Note that this means the height of the box is x. The two sides that would form the length and the width of the box would be the original 8 in sides minus the corners cut out, $8 - 2x$.

$$\begin{aligned} V &= \text{length} \cdot \text{width} \cdot \text{height} \\ &= (8-2x)(8-2x)(x) \\ &= (64 - 32x + 4x^2)(x) \\ &= 64x - 32x^2 + 4x^3 \end{aligned}$$

The volume in terms of x is $V(x) = 4x^3 - 32x^2 + 64x$.