1 Limits

1.2 Practice

1) The Strategy for Limits at Infinity of Rational Functions states we can divide each term by the highest power of x when we are trying to calculate the limit as x approaches ∞. Think about why this works. Does it work for limits as x approaches $-\infty$? Is this helpful if we are calculating the limit as x approaches 2?

2)

a) $\lim_{x\to 3} \frac{2x^2-5x-3}{x-3} = 7$

b) $\lim_{x\to\infty} (x^3-4x^2+5) = \infty$

c) $\lim_{x\to-\infty} (x^3-4x^2+5) = -\infty$

d) $\lim_{x\to 0} \left(\frac{5}{x^2+x} - \frac{5}{x}\right) = -5$

e) $\lim_{x\to 1/2} \frac{x^{-1}-2}{x-1/2} = -4$

f) $\lim_{x\to-\infty} \frac{2x^2}{x^2-4} = 2$

g) $\lim_{x\to\infty} \frac{x^3}{4x^2+3} = \infty$

h) $\lim_{x\to 2} \frac{x^4-16}{x-2} = 32$

i) $\lim_{x\to 0} |x-1| = 1$

j) $\lim_{x\to -3} |x-1| = 4$

k) $\lim_{x\to\infty} \frac{2x^3}{3x^2-4} = \infty$

l) $\lim_{x\to-\infty} \frac{x+2}{x^2+x+1} = 0$

m) $\lim_{x\to-\infty} \frac{3x^3}{3x^2-2} = -\infty$

n) $\lim_{x\to\infty} \frac{-3x^2}{4x+4} = -\infty$

o) $\lim_{x\to\infty} \frac{x+1}{2x^2+2x+1} = 0$

p) $\lim_{x\to 0} \sqrt{\cos(x)} = 1$

q) $\lim_{x\to 1} e^{1/x} = e$

r) $\lim_{x\to\pi/2} \tan(x^2) = \tan\left(\frac{\pi^2}{4}\right) \approx -0.799$

s) $\lim_{x\to-\infty} \frac{5x^2+2}{3x^3-2x} = 0$

t) $\lim_{x\to -2} \frac{\frac{1}{2}+\frac{1}{x}}{x+2} = -\frac{1}{4}$

u) $\lim_{x\to -1} \frac{x^4+3x^3-x^2+x+4}{x+1} = 8$

3)

a) $\lim_{x\to 2} f(x)$ for $f(x) = \begin{cases} \frac{x^2-4}{x-2} & \text{for } x \neq 2 \\ 2 & \text{for } x = 2 \end{cases}$ Limit DNE

b) $\lim_{x\to 2} f(x)$ for $f(x) = \begin{cases} x^2+5 & \text{for } x \leq 2 \\ -x+11 & \text{for } x > 2 \end{cases}$ Limit $= 9$

c) $\lim_{x\to 1} f(x)$ for $f(x) = \begin{cases} \ln(x) & \text{for } 0 < x \leq 1 \\ \cos(x) & \text{for } x > 1 \end{cases}$ Limit DNE

d) $\lim_{x\to -1^+} f(x)$ for $f(x) = \begin{cases} x+2 & \text{for } x \leq -1 \\ \frac{1}{x} & \text{for } x > -1 \end{cases}$ Limit $= -1$

2 Asymptotes

2.1 Practice

1) This question should help clarify the difference between vertical and horizontal asymptotes.

   a) Does a function ever cross its vertical asymptote? No

   b) Does a function ever cross its horizontal asymptote? Hint: Consider the horizontal asymptote and graph of $f(x) = \frac{x+2}{x^2+1}$. Yes

   c) Which type of asymptote corresponds to the domain of the function? Vertical

2) Find the vertical asymptote(s), if any, of the following:

   a) $f(x) = \frac{1}{x^2-1}$ has a VA at $x = 1$ and $x = -1$

   b) $f(x) = \tan(x)$ has a VA at $x = \frac{\pi}{2}$, $x = -\frac{\pi}{2}$ and at $x = \pm\frac{c\pi}{2}$ where c is odd

   c) $f(x) = \csc(x)$ has a VA at $x = 0$, $x = \pi$ and at $x = c\pi$ for any integer c

   d) $f(x) = \ln(x)$ has a VA at $x = 0$

3) Find the horizontal asymptote(s), if any, of the following:

   a) $f(x) = \sin(x)$ has no HA

   b) $f(x) = e^x$ has a HA at $y = 0$

   c) $f(x) = \frac{x-1}{\sqrt{x}}$ has no HA

   d) $f(x) = \frac{3x^3+4}{x^3-x+2}$ has a HA at $y = 3$

3 Continuity

3.1 Practice

1)

a) $f(x) = \sqrt{x}$ at $x = -1$ and at $x = 4$
$f(x) = \sqrt{x}$ is not continuous at $x = -1$ and is continuous at $x = 4$

b) $f(x) = \dfrac{x^2 - 4}{x - 2}$ at $x = 2$
$f(x) = \dfrac{x^2 - 4}{x - 2}$ is not continuous at $x = 2$

c) $f(x) = \begin{cases} \dfrac{x^2 - 1}{x + 1} & \text{for } x < -1 \\ x & \text{for } -1 \leq x \leq 2 \\ -x + 4 & \text{for } x > 2 \end{cases}$ at $x = -1$ and at $x = 2$
$f(x)$ is not continuous at $x = -1$ and is continuous at $x = 2$

d) $f(x) = e^x$ on the entire real line
$f(x) = e^x$ is continuous on the entire real line

e) $f(x) = \ln(x)$ on the entire real line
$f(x) = \ln(x)$ is not continuous on the entire real line

f) $f(x) = \ln(x^3)$ at $x = -1$
$f(x) = \ln(x^3)$ is not continuous at $x = -1$

g) $f(x) = \tan\sqrt{x}$ at $x = \dfrac{\pi^2}{4}$
$f(x) = \tan(\sqrt{x})$ is not continuous at $x = \dfrac{\pi^2}{4}$

h) $f(x) = \tan(\cos(x))$ on the entire real line
$f(x) = \tan(\cos(x))$ is continuous on the entire real line

2) Find the a and/or b value such that $f(x)$ is continuous on the entire real line:

a) $f(x) = \begin{cases} \dfrac{x^2 - 9}{x - 3} & \text{for } x \neq 3 \\ a & \text{for } x = 3 \end{cases}$ $a = 6$

b) $f(x) = \begin{cases} 2 & \text{for } x \leq -1 \\ ax + b & \text{for } -1 < x < 3 \\ a - 2x & \text{for } x \geq 3 \end{cases}$ $a = -\dfrac{8}{3}$, $b = -\dfrac{2}{3}$

c) $f(x) = \begin{cases} \cos(x) & \text{for } x \leq \dfrac{\pi}{2} \\ \sin(x) + a & \text{for } x > \dfrac{\pi}{2} \end{cases}$ $a = -1$

d) $f(x) = \begin{cases} x^3 & \text{for } x \leq 2 \\ ax^2 & \text{for } x > 2 \end{cases}$ $a = 2$

4 Squeeze Theorem

4.1 Practice

1) This problem is meant to walk you through how to approach Squeeze Theorem problems.

   a) Consider $\lim_{x \to 0^-} x^3 \cos\left(\frac{2}{x}\right)$.

   b) Which function can we easily bound, $\cos\left(\frac{2}{x}\right)$ or x^3? [Hint: Think of the graphs of the functions.]

   c) Using that
   $$-1 \leq \cos\left(\frac{2}{x}\right) \leq +1,$$
   how do we get an inequality for $x^3 \cos\left(\frac{2}{x}\right)$ [Hint: Multiply the inequality by x^3.]

   d) Since we are looking at the limit at $x \to 0^-$, we know $x < 0$, so $x^3 < 0$. That means the inequality signs switch directions, so we get
   $$-x^3 \leq x^3 \cos\left(\frac{2}{x}\right) \leq x^3.$$

   e) We know how to find the limit of x^3 and $-x^3$ as $x \to 0^-$, so now we apply the Squeeze Theorem to get our final answer.
   $$\lim_{x \to 0^-} x^3 \cos\left(\frac{2}{x}\right) = 0$$

2) Compute the following limits using the Squeeze Theorem.

   a) $\lim_{x \to \infty} \frac{\sin(x)}{x} = 0$ b) $\lim_{x \to \infty} \frac{2 - \cos(x)}{x + 3} = 0$ c) $\lim_{x \to \infty} \frac{\cos^2(2x)}{3 - 2x} = 0$

5 Intermediate Value Theorem

5.1 Practice

1) For each function, does the Intermediate Value Theorem apply for the given interval?

   a) $f(x) = 3x^2 + 2x - 1$ on $[0, 4]$ — Yes

   b) $f(x) = \tan(x)$ on $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$ — Yes

   c) $f(x) = \cos(x)$ on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ — Yes

   d) $f(x) = \tan(x)$ on $[0, \pi]$ — No

2) For each function, find the c value guaranteed by the Intermediate Value Theorem:

   a) $f(x) = 3x^2 + 2x - 1$ on $[0, 4]$, $f(c) = 15$ — $c = 2$

   b) $f(x) = \frac{x^2 + x}{x - 1}$ on $\left[\frac{5}{2}, 4\right]$, $f(c) = 6$ — $c = 3$

6 Instantaneous Velocity

6.1 Practice

a) A ball is thrown vertically into the air from ground level with an initial velocity of 15 m/s. Its height at time t is $s(t) = 15t - 4.8t^2$. What is the instantaneous velocity at $t = 1$? — $v(1) = 5.4\ m/s$

b) Let $p(t) = t^3 - 45t$ denote the distance (in meters) to the right of the origin of a particle at time t minutes after noon. Find the instantaneous velocity of the particle at 12:02 *pm*. — $v(2) = -33\ m/min$

c) A particle is moving along a straight line so that its position at time t seconds is given by $s(t) = 4t^2 - t$ meters. Find the instantaneous velocity at $t = 2$. — $v(2) = 15\ m/s$

1 Derivatives

1.3 Practice

1) Use the definition of the derivative to differentiate:

a) $f(x) = x^2 + 7$
$f'(x) = 2x$

b) $f(x) = \sqrt{x-1}$
$f'(x) = \dfrac{1}{2\sqrt{x-1}}$

c) $f(x) = ax^2 + c$
$f'(x) = 2ax$

d) $f(x) = x^2 - 5x + 2$
$f'(x) = 2x - 5$

e) $f(x) = \dfrac{1}{x}$
$f'(x) = -\dfrac{1}{x^2}$

f) $f(x) = \dfrac{1}{x-2}$
$f'(x) = -\dfrac{1}{(x-2)^2}$

g) $f(x) = \sqrt{x^2-1}$
$f'(x) = \dfrac{x}{\sqrt{x^2-1}}$

h) $f(x) = ax^3$
$f'(x) = 3ax^2$

2) Use any method/rule to find the derivative:

a) $f(x) = e^x \sin(x)$ $\quad f'(x) = e^x \cos(x) + e^x \sin(x)$

b) $f(x) = (x^4 + 3x)^{-1}$ $\quad f'(x) = -1(x^4 + 3x)^{-2}(4x^3 + 3)$

c) $f(x) = 3x^2(x^3 + 1)^7$ $\quad f'(x) = 63x^4(x^3 + 1)^6 + 6x(x^3 + 1)^7$

d) $f(x) = \cos^4(x) - 2x^2$ $\quad f'(x) = -4\cos^3(x)\sin(x) - 4x$

e) $f(x) = \dfrac{x}{1+x^2}$ $\quad f'(x) = \dfrac{(1+x^2) - 2x^2}{(1+x^2)^2}$

f) $f(x) = \dfrac{x^2-1}{x}$ $\quad f'(x) = 1 + x^{-2}$

g) $f(x) = 3x^2\sqrt{x}$ $\quad f'(x) = \dfrac{15}{2}x^{3/2}$

h) $f(x) = \ln(xe^{7x})$ $\quad f'(x) = \dfrac{1}{x} + 7$

i) $f(x) = \dfrac{2x^4 + 3x^2 - 1}{x^2}$ $\quad f'(x) = 4x + 2x^{-3}$

j) $f(x) = \dfrac{4(3x-1)^2}{x^2 + 7^x}$ $\quad f'(x) = \dfrac{(x^2 + 7^x)(72x - 24) - 4(3x-1)^2(2x + 7^x \ln(7))}{(x^2 + 7^x)^2}$

k) $f(x) = 2x - \dfrac{4}{\sqrt{x}}$ $\quad f'(x) = 2 + 2x^{-3/2}$

l) $f(x) = \dfrac{x}{\sqrt{1 - (\ln(x))^2}}$ $\quad f'(x) = \dfrac{(1 - (\ln(x))^2)^{1/2} + (1 - (\ln(x))^2)^{-1/2}(\ln(x))}{1 - (\ln(x))^2}$

m) $f(x) = x^3\sqrt[5]{7-x}$ $\qquad f'(x) = -\frac{x^3}{5}(7-x)^{-4/5} + 3x^2(7-x)^{1/5}$

n) $f(x) = (xe^x)^\pi$ $\qquad f'(x) = \pi(xe^x)^{\pi-1}(xe^x + e^x)$

o) $f(x) = [\arctan(2x)]^{10}$ $\qquad f'(x) = 10\,[\arctan(2x)]^9 \ \frac{2}{1+(2x)^2}$

p) $f(x) = (e^{2x} + e)^{1/2}$ $\qquad f'(x) = (e^{2x} + e)^{-1/2}(e^{2x})$

q) $f(x) = \left(7x + \sqrt{x^2+3}\right)^6$ $\qquad f'(x) = 6\left(7x + \sqrt{x^2+3}\right)^5 \left(7 + x(x^2+3)^{-1/2}\right)$

r) $f(x) = e^x(x^2+3)(x^3+4)$ $\qquad f'(x) = [3x^2e^x(x^2+3)] + (x^3+4)[(2xe^x + (x^2+3)e^x]$

s) $f(x) = \sqrt{\frac{2x+5}{7x-9}}$ $\qquad f'(x) = \frac{1}{2}\left(\frac{2x+5}{7x-9}\right)^{-1/2} \frac{(14x-18)-(14x+35)}{(7x-9)^2}$

t) $f(x) = \frac{\frac{1}{x} + \frac{1}{x^2}}{x-1}$ $\qquad f'(x) = \frac{(x-1)(-x^{-2} - 2x^{-3}) - (x^{-1} + x^{-2})}{(x-1)^2}$

u) $f(x) = [\ln(5x^2+9)]^3$ $\qquad f'(x) = 3\,[\ln(5x^2+9)]^2 \ \frac{10x}{5x^2+9}$

v) $f(x) = \sec(x)\sin(3x)$ $\qquad f'(x) = 3\sec(x)\cos(3x) + \sin(3x)\sec(x)\tan(x)$

w) $f(x) = \tan(\cos(x))$ $\qquad f'(x) = -\sin(x)\sec^2(\cos(x))$

x) $f(x) = \log_5(3x^2+4x)$ $\qquad f'(x) = \frac{6x+4}{(3x^2+4x)\ln 5}$

y) $f(x) = e^{5\tan(x)}$ $\qquad f'(x) = e^{5\tan(x)}(5\sec^2(x))$

z) $f(x) = \arcsin(x^3+2)$ $\qquad f'(x) = \frac{3x^2}{\sqrt{1-(x^3+2)^2}}$

3) Find the equation of the tangent line to $f(x) = \frac{1}{2}x^5 - 2x$ at

a) $x = 0$
$y = -2x$

b) $x = 1$
$y = \frac{1}{2}x - 2$

c) $x = 2$
$y = 38x - 64$

4) Find the equation of the tangent line to $f(x) = e^{2x^3}$ at

a) $x = 0$
$y = 1$

b) $x = 1$
$y = 6e^2x - 5e^2$

c) $x = -2$
$y = 24e^{-16}x + 49e^{-16}$

5) Suppose the position of an object is given by $s(t) = x^4 - 3x^2 - x + 1$ in m, where time is measured in s.

   a) Find the velocity $v(t)$ and $v(2)$.
   $v(t) = 4x^3 - 6x - 1$ in m/s, $v(2) = 19\ m/s$

   b) Find the acceleration $a(t)$ and $a(2)$.
   $a(t) = 12x^2 - 6$ in m/s^2, $a(2) = 42\ m/s^2$

6) Suppose the height of a coin dropped off a 100 ft tall bridge is given by $s(t) = 100 - t^2$ in ft, where time is measured in s.

   a) How long does it take for the coin to hit the water?
   $s(t) = 0$ when $t = 10\ s$

   b) Find the velocity $v(t)$.
   $v(t) = -2t$ in ft/s

   c) What is the coin's velocity when it hits the water?
   $v(10) = -20\ ft/s$

2 Derivative Techniques

2.1 Practice

1) Differentiate the following using implicit and/or logarithmic differentiation.

   a) $3y = xe^{5y}$ $\quad \dfrac{dy}{dx} = \dfrac{e^{5y}}{3 - 5xe^{5y}}$

   b) $xy + y^2 + x^3 = 7$ $\quad \dfrac{dy}{dx} = \dfrac{-3x^2 - y}{x + 2y}$

   c) $\dfrac{\sin(y)}{y^2 + 1} = 3x$ $\quad \dfrac{dy}{dx} = \dfrac{3(y^2 + 1)^2}{(y^2 + 1)\cos(y) - 2y\sin(y)}$

   d) $2x^3 = 2y^2 + 5$ $\quad \dfrac{dy}{dx} = \dfrac{3x^2}{2y}$

   e) $3x^2 + 3y^2 = 2$ $\quad \dfrac{dy}{dx} = -\dfrac{x}{y}$

   f) $5y^2 = 2x^3 - 5y$ $\quad \dfrac{dy}{dx} = \dfrac{6x^2}{10y + 5}$

g) $4x^2 = 2y^3 + 4y$ $$\frac{dy}{dx} = \frac{4x}{3y^2+2}$$

h) $3x^2y^2 = 4x^2 - 4xy$ $$\frac{dy}{dx} = \frac{4x - 2y - 3xy^2}{3x^2y + 2x}$$

i) $5x^3 + xy^2 = 5x^3y^3$ $$\frac{dy}{dx} = \frac{15x^2y^3 - 15x^2 - y^2}{2xy - 15x^3y^2}$$

j) $2x^3 = (3xy+1)^2$ $$\frac{dy}{dx} = \frac{x}{3xy+1} - \frac{y}{x}$$

k) $x^2 = (4x^2y^3+1)^2$ $$\frac{dy}{dx} = \frac{1}{12xy^2(4x^2y^3+1)} - \frac{2y}{3x}$$

l) $\sin(2x^2y^3) = 3x^3 + 1$ $$\frac{dy}{dx} = \frac{9x - 4y^3\cos(2x^2y^2)}{6xy^2\cos(2x^2y^3)}$$

m) $3x^2 + 3 = \ln(5xy^2)$ $$\frac{dy}{dx} = \frac{6x^2y^2 - y^2}{2xy}$$

n) $y = 2x^{2x}$ $$\frac{dy}{dx} = (2\ln x + 2)2x^{2x}$$

o) $y = 5x^{5x}$ $$\frac{dy}{dx} = (5\ln x + 5)5x^{5x}$$

p) $y = 4x^{x^4}$ $$\frac{dy}{dx} = (4x^3\ln x + x^3)4x^{x^4}$$

q) $y = (3x^4+4)^3\sqrt{5x^3+1}$ $$\frac{dy}{dx} = \left[\frac{36x^3}{3x^4+4} + \frac{15x^2}{2(5x^3+1)}\right](3x^4+4)^3\sqrt{5x^3+1}$$

r) $y = \frac{(3x^4-2)^5}{(3x^3+4)^2}$ $$\frac{dy}{dx} = \left[\frac{60x^3}{3x^4-2} - \frac{18x^2}{3x^3+4}\right]\frac{(3x^4-2)^5}{(3x^3+4)^2}$$

s) $y = \sqrt{3x^2+1}\,(3x^4+1)^3$ $\quad \dfrac{dy}{dx} = \left[\dfrac{3x}{3x^2+1} + \dfrac{36x^3}{3x^4+1}\right]\sqrt{3x^2+1}\,(3x^4+1)^3$

t) $y = \dfrac{\sqrt{2x^3+3}}{(x^4-3)^3}$ $\quad \dfrac{dy}{dx} = \left[\dfrac{3x^2}{2x^3+3} - \dfrac{12x^3}{x^4-3}\right]\dfrac{\sqrt{2x^3+3}}{(x^4-3)^3}$

u) $y = (2x^2-5)^3\sqrt{x^2-2}$ $\quad \dfrac{dy}{dx} = \left[\dfrac{12x}{2x^2-5} + \dfrac{x}{x^2-2}\right](2x^2-5)^3\sqrt{x^2-2}$

v) $y = \dfrac{(5x-4)^4}{(3x^2+5)^5\,(5x^4-3)^3}$ $\quad \dfrac{dy}{dx} = \left[\dfrac{20}{5x-4} - \dfrac{30x}{3x^2+5} - \dfrac{60x^3}{5x^4-3}\right]\dfrac{(5x-4)^4}{(3x^2+5)^5\,(5x^4-3)^3}$

2) Find the equation of the tangent line to $x^2y^2 - 2x = 4 - y$ at the point $(2, -2)$.
$y + 2 = \dfrac{14}{15}(x-2)$

3) Find all points where the tangent line to $x^2 + y^3 - 3y = 4$ is horizontal and where it is vertical.
The tangent line is horizontal when $x = 0$ and vertical when $y = 1$ or $y = -1$.

3 Derivative Applications

3.1 Practice

1) Water leaking onto a floor forms a circular pool. The radius of the pool increases at a rate of 4 cm/min. How fast is the area of the pool increasing when the radius is 5 cm?

$$\frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt} = 40\pi\ cm^2/min$$

2) Oil spilling from a ruptured tanker spreads in a circle on the surface of the ocean. The area of the spill increases at a rate of $9\pi\ m^2/min$. How fast is the radius of the spill increasing when the radius is 10 m?

$$\frac{dr}{dt} = \frac{1}{2\pi r} \cdot \frac{dA}{dt} = \frac{9}{20}\ m/min$$

3) A conical paper cup is 10 cm tall with a radius of 10 cm. The cup is being filled with water so that the water level rises at a rate of 2 cm/sec. At what rate is the water being poured into the cup when the water level is 8 cm?

$$\frac{dV}{dt} = \pi h^2 \cdot \frac{dh}{dt} = 128\pi\ cm^3/sec$$

4) A spherical balloon is inflated so that its radius, r, increases at a rate of $\frac{2}{r}$ cm/sec. How fast is the volume of the balloon increasing when the radius is 4 cm?

$$\frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt} = 32\pi\ cm^3/sec$$

5) An observer stands 700 ft away from a launch pad to observe a rocket launch. The rocket blasts off and maintains a velocity of 900 ft/sec. Assume the scenario can be modeled as a right triangle. How fast is the observer to rocket distance changing when the rocket is 2400 ft from the ground?

Let a = altitude of rocket, and z = distance from observer to rocket.

$$\frac{dz}{dt} = \frac{a}{z} \cdot \frac{da}{dt} = 864\ ft/sec$$

6) An 8 ft ladder is leaning against a wall. The top of the ladder is sliding down the wall at the rate of 2 ft/sec. How fast is the bottom of the ladder moving along the ground at the point in time when the bottom of the ladder is 4 ft from the wall?

$$\frac{dx}{dt} = 2\sqrt{3}\ ft/sec$$

7) The width of a rectangle is increasing at a rate of 2 cm/sec, while the length increases at 3 cm/sec. At what rate is the area increasing when $w = 4$ cm and $l = 5$ cm?

$$\frac{dA}{dt} = 22\ cm^2/sec$$

1 Linearization

1.1 Practice

1) Use linearization to approximate $y = \sqrt{4+\sin(x)}$ at $x = 0.12$, using the tangent line at $x = 0$.
$\sqrt{4+\sin(.12)} \approx 2.03$

2) Use linearization to approximate $\sin(.2)$. What is an appropriate c value to use?
Use $c = 0$, $\sin(.2) \approx .2$.

3) Use linearization to approximate $f(5.02)$ if $f(x) = \dfrac{1}{\sqrt{4+x}}$.
Use $c = 5$, $f(5.02) \approx .333$.

4) Use linearization to approximate the value of $(2.003)^4$.
Use $c = 2$ and $f(x) = x^4$, $f(2.003) \approx 16.096$.

2 Newton's Method

2.1 Practice

1) Use Newton's Method to find a solution to $x^2 - 17 = 0$.
$x_1 = 4$, $x_2 = 4.125$, $x_3 = 4.123$ or
$x_1 = -4$, $x_2 = -4.125$, $x_3 = -4.123$

2) Find the solution to $\cos(x) = x$ (make a sketch to help with your initial guess).
$x_1 = 1$, $x_2 = 0.7504$, $x_3 = 0.7391$

3) Use Newton's Method to find $\ln(2)$. (Hint: Start by finding an equation whose solution is $\ln(2)$). What happens if you start with the initial guess -4?
Use $f(x) = e^x - 2$.
$x_1 = 1$, $x_2 = 0.7358$, $x_3 = 0.69404$

4) Use Newton's method to approximate $\sqrt[100]{100}$ to four decimal places.
Use $f(x) = x^{100} - 100$.
$x_1 = 1, x_2 = 1.99, x_3 = 1.9701, x_4 = 1.9504$

5) Use Newton's method to find the roots of $\dfrac{1}{x} = 1 + x^3$ to 3 decimal places.
$x_1 = 1, x_2 = 0.75, x_3 = 0.7244, x_4 = 0.7245$ and
$x_1 = -1, x_2 = -1.25, x_3 = -1.2213, x_4 = -1.2207$, $x_5 = -1.2207$

6) The equation $x^2 = 2^x$ has two integer solutions $x = 2$ and $x = 4$. Use Newton's method to approximate the other solution. Graph the function to inspire your first estimate.
$x_1 = -1$, $x_2 = -0.7869$, $x_3 = -0.7668$, $x_4 = -0.7667$

3 Mean Value Theorem

3.1 Practice

1) For each problem, find the values of c that satisfy the Mean Value Theorem.

a) $y = -x^2 + 8x - 17; \quad [3, 6]$
$c = \dfrac{9}{2}$

b) $y = x^3 - 9x^2 + 24x - 18; \quad [2, 4]$
$c = \dfrac{9 \pm \sqrt{3}}{3}$

c) $y = -\dfrac{x^2}{2} + x - \dfrac{1}{2}; \quad [-2, 1]$
$c = -\dfrac{1}{2}$

d) $y = \dfrac{x^2}{2} - 2x - 1; \quad [-1, 1]$
$c = 0$

e) $y = x^3 + 3x^2 - 2; \quad [-2, 0]$
$c = \dfrac{-3 \pm \sqrt{3}}{3}$

f) $y = \dfrac{x^2}{2x - 4}; \quad [-4, 1]$
$c = 2 - \sqrt{6}$

2) For each problem, determine if the Mean Value Theorem can be applied. If it can, find all values of c that satisfy the theorem. If it cannot, explain why.

a) $y = -\dfrac{x^2}{4x + 8}; \quad [-3, -1]$
MVT cannot be applied, since y is not continuous on $[-3, -1]$

b) $y = -(6x + 24)^{2/3}; \quad [-4, -1]$
$c = -\dfrac{28}{9}$

c) $y = \dfrac{-x^2 + 9}{4x}; \quad [1, 3]$
$c = \sqrt{3}$

d) $y = (x - 3)^{2/3}; \quad [1, 4]$
MVT cannot be applied, since y is not differentiable on $(1, 4)$

4 Extrema

4.1 Practice

1) For each problem, find all relative maxima and minima, intervals where y is increasing/decreasing, and intervals where y is concave up/down.

a) $y = -x^3 - 3x^2 - 1$
Min: $(-2, -5)$, Max: $(0, -1)$
Increasing: $[-2, 0]$
Decreasing: $(-\infty, -2]$, $[0, \infty)$
Concave Up: $(-\infty, -1)$
Concave Down: $(-1, \infty)$

b) $y = x^4 - 2x^2 + 3$
Min: $(\pm 1, 2)$, Max: $(0, 3)$
Increasing: $[-1, 0]$, $[1, \infty)$
Decreasing: $(-\infty, -1]$, $[0, 1]$
Concave Up: $\left(-\infty, -\dfrac{1}{\sqrt{3}}\right), \left(\dfrac{1}{\sqrt{3}}, \infty\right)$
Concave Down: $\left(-\dfrac{1}{\sqrt{3}}, \dfrac{1}{\sqrt{3}}\right)$

c) $y = x^3 - 6x^2 + 9x + 1$
Min: $(3, 1)$, Max: $(1, 5)$
Increasing: $(-\infty, 1]$, $[3, \infty)$
Decreasing: $[1, 3]$
Concave Up: $(2, \infty)$
Concave Down: $(-\infty, 2)$

d) $y = \dfrac{1}{x^3 - 3x^2}$
Max: $(2, -0.25)$
Increasing: $(0, 2]$
Decreasing: $(-\infty, 0)$, $[2, 3)$, $(3, \infty)$
Concave Up: $(3, \infty)$
Concave Down: $(-\infty, 0)$, $(0, 3)$

2) For each problem, find the absolute maxima and minima on the given closed interval.

a) $y = -x^3 - 6x^2 - 9x + 3; \quad [-3, -1]$
Min: $(-3, 3)$, Max: $(-1, 7)$

b) $y = \dfrac{8}{x^2 + 4}; \quad [0, 5]$
Min: $\left(5, \dfrac{8}{29}\right)$, Max: $(0, 2)$

c) $y = \dfrac{x^2}{3x - 6}; \quad [3, 6]$
Min: $\left(4, \dfrac{8}{3}\right)$, Max: $(3, 3), (6, 3)$

d) $y = (5x + 25)^{1/3}; \quad [-2, 2]$
Min: $(-2, \sqrt[3]{15})$, Max: $(2, \sqrt[3]{35})$

e) $y = x^3 - 3x^2 + 6; \quad [0, \infty)$
Min: $(2, 2)$, Max: None

f) $y = \dfrac{4}{x^2 + 2}; \quad (-5, -2]$
Min: None, Max: $\left(-2, \dfrac{2}{3}\right)$

g) $y = x^3 - 3x^2 - 3; \quad (0, 3)$
Min: $(2, -7)$, Max: None

h) $y = x^4 - 2x^2 - 3; \quad (0, \infty)$
Min: $(1, -4)$, Max: None

5 Optimization

5.1 Practice

1) The sum of two positive numbers is 48. What is the smallest possible value for the sum of their squares?
The sum of squares is minimized when $x = y = 24$ and the sum is 1152.

2) A rectangular storage container with an open top is to have a volume of 10 m^3. The length of its base is twice the width. Material for the base costs \$10 per square meter. Material for the sides costs \$6 per square meter. Find the costs of materials for the cheapest such container.
The cheapest container has $w = 1.651\ m$, $l = 3.302\ m$, $h = 1.834\ m$, and cost \$163.54.

3) A farmer is planting a small orchard. He is advised that, if he plants up to 60 trees of a particular type on his plot of land the average harvest from each tree will be about 120 kg, but for each additional tree planted the expected yield will go down by an average of 2 kg per tree as a result of overcrowding. How many trees should he plant for the maximum yield of fruit?
He should plant 60 trees.

4) Suppose you had to use exactly 200 m of fencing to make either one square enclosure or two separate square enclosures of any size you wished. What plan would give you the least area? What plan would give you the greatest area?
The single enclosure gives the greatest area. The two separate enclosures gives the least area.

5) You want to enclose two adjacent rectangular pens using 200 ft of fencing. What dimensions should be used to maximize the enclosed area?
The combined length of the enclosures should be 50 ft and the width should be $\frac{100}{3}$ ft.

6) Find two positive numbers such that their product is 192 and the sum of the first plus three times the second is minimized.
The numbers are 24 and 8.

7) A wire 6 m long is cut into twelve pieces. These pieces are welded together at right angles to form the frame of a box with a square base. Where should the cuts be made to maximize the volume of the box? Where should the cuts be made to maximize the total surface area for the box?
The twelve pieces should each be 0.5 m to maximize both volume and surface area.

8) Four ft of wire is to be used to form a square and a circle. How much of the wire should be used for the square and how much should be used for the circle to enclose the maximum total area?
Total area is maximized when all 4 ft go toward the circle.

9) The combined perimeter of a circle and a square is 16 in. Find the dimensions of the circle and square that produce a minimum total area.
Total area is minimized when the radius of the circle is 1.12 in and the side of the square is 2.24 in.

10) Find two positive numbers such that the sum of the first and twice the second is 100 and their product is a maximum.
The numbers are 50 and 25.

6 Indeterminate Forms

6.1 Practice

1) Find the following limits:

a) $\lim_{x\to\infty} \frac{\ln(x)}{x-1} = 0$

b) $\lim_{x\to 1} \frac{x^2-1}{x^2-x} = 2$

c) $\lim_{x\to\infty} \frac{e^{x/10}}{x^3} = \infty$

d) $\lim_{x\to\pi^-} \frac{\sin(x)}{1-\cos(x)} = 0$

e) $\lim_{x\to-\infty} x^2e^x = 0$

f) $\lim_{x\to 0^+} (\csc(x) - \cot(x)) = 0$

g) $\lim_{x\to 0^+} x^x = 1$

h) $\lim_{x\to 0} \frac{\sin(4x)}{\tan(5x)} = \frac{4}{5}$

i) $\lim_{x\to 1} \frac{\ln(x)}{\sin(\pi x)} = -\frac{1}{\pi}$

j) $\lim_{x\to\infty} \sqrt{x^2+x} - x = \frac{1}{2}$

k) $\lim_{x\to\infty} x^{1/x} = 1$

l) $\lim_{x\to\infty} x\sin\left(\frac{\pi}{x}\right) = \pi$

m) $\lim_{x\to 0^+} \frac{2x+7}{4x+1} = 7$

n) $\lim_{x\to 0^+} \left(\cot(x) - \frac{1}{x}\right) = 0$

o) $\lim_{x\to 1} \frac{5\ln(x)}{x-1} = 5$

p) $\lim_{x\to\infty} \frac{e^x}{e^x+e^{-x}} = 1$

q) $\lim_{x\to 0} \frac{3x}{\ln(x+1)} = 3$

r) $\lim_{x\to 0^+} 5x^2\ln(x) = 0$

s) $\lim_{x\to\infty} 4xe^{-x} = 0$

t) $\lim_{x\to\frac{\pi}{2}} (3\sec(x) - 3\tan(x)) = 0$

u) $\lim_{x\to\infty} \left(\frac{x^2}{x-1} - \frac{x^2}{x+1}\right) = 2$

v) $\lim_{x\to 0^+} 5\,(\tan(x))^{\sin(x)} = 5$

w) $\lim_{x\to 0^+} 3x^x = 3$

x) $\lim_{x\to 0} \frac{\sin(x^3)}{\sin^3(x)} = 1$

2) Explain why l'Hopital's Rule will fail in the following two cases:

a) $\lim_{x\to 0} \frac{\sqrt{4x^2+3}}{x+3} = \frac{\sqrt{3}}{3}$, but if we use l'Hopital's Rule we get that this limit is 0.

b) $\lim_{x\to\infty} \frac{e^x-e^{-x}}{e^x+e^{-x}} = 1$, but if we use l'Hopital's Rule we get stuck in a loop.

7 Differentials

7.1 Practice

1) Let $f(x) = 4\sin(x)$. Find the differential of f and approximate the change in f as x changes from $\frac{\pi}{6}$ to $\frac{\pi}{6}+0.1$. Use this to approximate $f\left(\frac{\pi}{6}+0.1\right)$. Now use the differential to approximate the relative change as x changes from $\frac{\pi}{6}$ to $\frac{\pi}{6}+0.1$.
$f'(x)\,dx = 4\cos(x)\,dx$, $df = 0.346$, $f\left(\frac{\pi}{6}+0.1\right) \approx 2.346$, $\frac{df}{f(\pi/6)} = 0.173$

2) Use differentials to approximate the increase in surface area of a cube if the length of each edge is changed from 10 cm to 10.1 cm. What is the exact change in surface area?
Approx change in SA $= 12\ cm^2$, Exact change in SA $= 12.06\ cm^2$

3) Find the length of string you would need to wrap the string around the earth at the equator. Use 3960 mi as the radius of the earth. Use differentials to approximate how much extra string you would need if the radius were off by 1 ft.
Length of string needed $= 7920\pi\ mi$, Extra string needed $= \frac{\pi}{2640}\ mi$

8 Antiderivatives

8.1 Practice

1) Find the following antiderivatives:

a) $\int (4x^3 - 12x^2 + 6x - 1)\ dx = x^4 - 4x^3 + 3x^2 - x + C$

b) $\int (x^5 + 3x^4 - x^2 + 4)\ dx = \frac{1}{6}x^6 + \frac{3}{5}x^5 - \frac{1}{3}x^3 + 4x + C$

c) $\int 7\sin(x)\ dx = -7\cos(x) + C$

d) $\int -3\csc(x)\cot(x)\ dx = 3\csc(x) + C$

e) $\int (4e - 7x)\ dx = 4ex - \frac{7}{2}x^2 + C$

f) $\int\ d\theta = \theta + C$

g) $\int \frac{3}{x}\ dx = 3\ln|x| + C$

h) $\int \frac{2}{\sec(x)}\ dx = 2\sin(x) + C$

i) $\int y^2\sqrt[3]{y}\ dy = \frac{3}{10}y^{10/3} + C$

j) $\int \left(5\cos(x) - \sec^2(3x)\right)\ dx = 5\sin(x) - \frac{1}{3}\tan(3x) + C$

k) $\int \left(5x^6 + \csc(x)\cot(x)\right)\ dx = \frac{5}{7}x^7 - \csc(x) + C$

l) $\int -\frac{5}{\sqrt{1-x^2}}\ dx = -5\arcsin(x) + C$

m) $\int 10e^x\ dx = 10e^x + C$

n) $\int -\frac{6}{5x}\ dx = -\frac{6}{5}\ln|x| + C$

o) $\int \left(\sqrt{x} + \frac{1}{2\sqrt{x}}\right)\ dx = \frac{2}{3}x^{3/2} + x^{1/2} + C$

p) $\int \frac{9}{1+x^2}\ dx = 9\arctan(x) + C$

1 Areas and Riemann Sums

1.1 Practice

1) Evaluate the following sums:

a) $\sum_{x=1}^{2} 6x = 18$

b) $\sum_{i=0}^{2} (i+5) = 18$

c) $\sum_{j=1}^{2} 5j = 15$

d) $\sum_{l=3}^{5} (l^2 - 3) = 41$

e) $\sum_{k=1}^{n} (2k+5) = n^2 + 6n$

f) $\sum_{t=1}^{n} 8t^2 = \frac{4n(n+1)(2n+1)}{3}$

2) Approximate the area under the curve $f(x) = x^2 + 2$ on the interval $[-2, 1]$ using 6 right rectangles. $A \approx 8.375$

3) Approximate the area under the curve $f(x) = \sqrt{x+1}$ on the interval $[-1, 0]$ using 4 left rectangles. $A \approx 0.518$

4) Approximate the area under the curve $f(x) = x^3 - 3$ on the interval $[1, 3]$ using 5 left rectangles. $A \approx 9.12$

5) Find the exact area under the curve $f(x) = 3x + 3$ from $[0, 3]$ by taking the limit of the appropriate sum. $A = 22.5$

2 Integration

2.5 Finding Derivatives with Fundamental Theorem of Calculus

a) $f(x) = \int_1^x t^2\, dt$ $\quad f'(x) = x^2$

b) $g(x) = \int_{-2}^x \sin(t)\, dt$ $\quad g'(x) = \sin(x)$

c) $h(x) = \int_0^x e^t\, dt$ $\quad h'(x) = e^x$

d) $f(x) = \int_1^x \frac{1}{t^2}\, dt$ $\quad f'(x) = \frac{1}{x^2}$

e) $f(x) = \int_1^{x^2} t^2\, dt$ $\quad f'(x) = 2x(x^2)^2 = 2x^5$

f) $g(x) = \int_{-2}^{x^5} \sin(t)\, dt$ $\qquad g'(x) = 5x^4 \sin(x^5)$

g) $h(x) = \int_{0}^{\ln(x)} e^t\, dt$ $\qquad h'(x) = e^{\ln(x)} \cdot \frac{1}{x} = 1$

h) $f(x) = \int_{1}^{2x^3} \frac{1}{t^2}\, dt$ $\qquad f'(x) = \frac{1}{(2x^3)^2} \cdot (6x^2) = \frac{3}{2x^4}$

2.6 Indefinite Integral Practice

a) $\int 8e^{4x}(e^{4x}-4)^{1/5}\, dx = \frac{5}{3}(e^{4x}-4)^{6/5} + C$

b) $\int \frac{1}{\sqrt{25-x^2}}\, dx = \arcsin\left(\frac{x}{5}\right) + C$

c) $\int \frac{1}{3x+1}\, dx = \frac{1}{3}\ln|3x+1| + C$

d) $\int \frac{50x}{\sec(5x^2+5)}\, dx = 5\sin(5x^2+5) + C$

e) $\int \sin^2(3x)\cos(3x)\, dx = \frac{1}{9}\sin^3(3x) + C$

f) $\int x^4 \sin(x)\, dx = (-x^4 + 12x^2 - 24)\cos(x) + (4x^3 - 24x)\sin(x) + C$

g) $\int \arccos(x)\, dx = x\arccos(x) - \sqrt{1-x^2} + C$

h) $\int x\sqrt{2x-1}\, dx = \frac{1}{10}(2x+1)^{5/2} - \frac{1}{6}(2x-1)^{3/2} + C$

i) $\int \sqrt{x}\ln(x)\, dx = \frac{2}{3}x^{3/2}\ln|x| - \frac{4}{9}x^{3/2} + C$

j) $\int (-9x^2 + 10x)\, dx = -3x^3 + 5x^2 + C$

k) $\int \frac{1}{x\sqrt{x^2-81}}\, dx = \frac{1}{9}\arctan\left(\frac{\sqrt{x^2-81}}{9}\right) + C$

l) $\int \ln(x+3)\, dx = (x+3)\ln(x+3) - x + C$

m) $\int \sqrt{2x-1}\, dx = \frac{1}{3}(2x-1)^{3/2} + C$

n) $\int 9\sin(3x)\,dx = -3\cos(3x) + C$

o) $\int x^4 e^x\,dx = e^x(x^4 - 4x^3 + 1x^2 - 24x + 24) + C$

p) $\int x\sec^2(x)\,dx = x\tan(x) + \ln|\cos(x)| + C$

q) $\int e^{-x}\tan(e^{-x})\,dx = \ln|\cos(e^{-x})| + C$

r) $\int 4\sin\left(\frac{x}{3}\right)\,dx = -12\cos\left(\frac{x}{3}\right) + C$

s) $\int -5\cos(\pi x)\,dx = -\frac{5}{\pi}\cos(\pi x) + C$

t) $\int \frac{1}{16+x^2}\,dx = \frac{1}{4}\arctan\left(\frac{x}{4}\right) + C$

u) $\int 4\tan(4x)\sec^5(4x)\,dx = \frac{1}{5}\sec^5(4x) + C$

v) $\int x \cdot 2^x\,dx = \frac{2^x\ln(2) - 1}{(\ln(2))^2} + C$

w) $\int x^2\cos(3x)\,dx = \frac{1}{27}\left[(9x^2-2)\sin(3x) + 6x\cos(3x)\right] + C$

x) $\int \frac{x}{\sqrt{25-x^2}}\,dx = -\sqrt{25-x^2} + C$

2.7 Definite Integral Practice

a) $\displaystyle\int_1^5 \frac{x}{\sqrt{2x-1}}\,dx = \frac{16}{3}$

b) $\displaystyle\int_0^1 \frac{10x^4}{9+4x^{10}}\,dx = \frac{1}{3}\arctan\left(\frac{2}{3}\right)$

c) $\displaystyle\int_0^1 \frac{x^3}{\sqrt{x^2+1}}\,dx = \frac{2}{3} - \frac{\sqrt{2}}{3}$

d) $\displaystyle\int_1^2 \frac{\ln(x)}{x^2}\,dx = \frac{1}{2} - \frac{\ln(2)}{2}$

e) $\displaystyle\int_0^{\pi/2} \frac{20x^3}{\sqrt{25-25x^8}}\,dx = \arcsin\left(\frac{\pi^4}{16}\right)$

f) $\displaystyle\int_{-3}^{-2} 5\sqrt[3]{2x+4}\,dx = -\frac{15\sqrt[3]{2}}{4}$

g) $\displaystyle\int_0^4 \frac{1}{\sqrt{2x-1}}\,dx = \sqrt{7} - i$

h) $\displaystyle\int_{-1}^2 \frac{2}{(2x+4)^3}\,dx = \frac{15}{128}$

i) $\displaystyle\int_{-1}^1 x(x^2+1)^3\,dx = 0$

j) $\displaystyle\int_0^{\pi/2} x\cos(x)\,dx = \frac{\pi}{2} - 1$

k) $\displaystyle\int_1^9 (x^{3/2}+2x+3)\,dx = \frac{1004}{5}$

l) $\displaystyle\int_0^{\pi} 20x\sin(5x^2-3)\,dx$
$= 2\cos(-3) - 2\cos(5\pi^2-3)$

m) $\displaystyle\int_1^2 \frac{2}{x(\ln(4x)-1)}\,dx$
$= 2\ln(\ln(8)-1) - 2\ln(\ln(4)-1)$

n) $\displaystyle\int_4^9 \frac{1}{3\sqrt{x}+\sqrt{x}}\,dx = 0.5$

o) $\displaystyle\int_0^1 \frac{20e^{5x}}{e^{5x}+3}\,dx = 4\ln(e^5+3) - 4\ln(4)$

p) $\displaystyle\int_0^{\pi/2} x\sin(2x)\,dx = \frac{\pi}{4}$

q) $\displaystyle\int_{-2}^1 (2t^2-1)^2\,dt = \frac{87}{5}$

r) $\displaystyle\int_{-1}^1 e^{2x-2}\,dx = \frac{1}{2} - \frac{1}{2e^4}$

s) $\displaystyle\int_{-4}^{-2} (|-3x-9| - x)\,dx = 9$

t) $\displaystyle\int_{-5}^1 -|x^2+4x|\,dx = -\frac{46}{3}$

u) $\displaystyle\int_2^4 x^2e^{2x}\,dx = \frac{25}{4}e^8 - \frac{5}{4}e^4$

v) $\displaystyle\int_{-1}^0 18x^2(3x^3+3)^2\,dx = 18$

w) $\displaystyle\int_{-2}^{-1} \frac{4}{x^2}\,dx = 2$

x) $\displaystyle\int_0^{\pi} \frac{-4\sin(4x)}{\sqrt{9-\cos^2(4x)}}\,dx = 0$

y) $\displaystyle\int_0^1 \frac{24x}{(4x^2+4)^2}\,dx = \frac{3}{8}$

z) $\displaystyle\int_0^5 xe^{-x}\,dx = 1 - \frac{6}{e^5}$

1 Area Between Curves

1.1 Practice

1) Find the area of the region bounded by $y = 4x - x^2$ and $y = 3$.
$A = \frac{4}{3}$

2) Find the area of the region bounded by $y = x^2$ and $y = 2x^2 - 25$.
$A = \frac{500}{3}$

3) Find the area of the region bounded by $y = 7x - 2x^2$ and $y = 3x$.
$A = \frac{8}{3}$

4) Find the area of the region bounded by $y = x(x-3)(x+3)$ and $y = 7x$.
$A = 128$

5) Find the area of the region bounded by $y = x^3 - 1$ and $y = x^2 - 1$.
$A = \frac{1}{12}$

6) Find the area of the region bounded by $x = 2y^2 + 12y + 19$, $x = -\frac{y^2}{2} - 4y - 10$, $y = -3$ and $y = -2$.
$A = \frac{29}{6}$

7) Find the area of the region bounded by $y = -2\sec^2(x)$, $y = 2\cos(x)$, $x = 0$ and $x = \frac{\pi}{4}$.
$A = \sqrt{2} + 2$

8) Find the area of the region bounded by $y = 2x^{\frac{2}{3}}$ and $y = x$.
$A = \frac{32}{5}$

9) Find the area of the region bounded by $y = -x^3 + 6x$ and $y = -x^2$.
$A = \frac{253}{12}$

10) Find the area of the region bounded by $y = -\frac{x^3}{2} + 2x^2$ and $y = -x^2 + 4x$.
$A = 4$

2 Volumes of Solids of Revolution

2.1 Practice

1) Find the volume of the solid resulting from rotating the region enclosed by $y = \sqrt{\sin(x)}$, the x-axis, $x = 0$ and $x = \pi$ around the x-axis.
   $A = 2\pi$

2) Find the volume of the solid resulting from rotating the region enclosed by $y = x^2$, $y = 4$, and the y-axis around the y-axis.
   $A = 8\pi$

3) Find the volume of the solid resulting from rotating the region enclosed by $y = 2 - x^2$ and $y = 1$ around the axis $y = 1$.
   $A = \dfrac{16\pi}{15}$

4) Find the volume of the solid resulting from rotating the region enclosed by $x = -y^2 + 2$ and $y = x$ around the axis $x = -2$.
   $A = \dfrac{108\pi}{5}$

5) Find the volume of the solid resulting from rotating the region enclosed by $y = \sqrt{x} + 1$ and $y = x^2 + 1$ around the axis $y = -1$.
   $A = \dfrac{49\pi}{30}$

6) Find the volume of the solid resulting from rotating the region enclosed by $y = \sqrt{x}$, $y = 0$ and $x = 4$ around the following axes:

   a) x-axis
      $A = 8\pi$

   b) y-axis
      $A = \dfrac{128\pi}{5}$

   c) $x = 4$
      $A = \dfrac{256\pi}{15}$

   d) $x = 6$
      $A = \dfrac{192\pi}{5}$

7) Use the disk method to derive the formula for the volume of a sphere with radius r.
   $V = \displaystyle\int_{-r}^{r} \pi(\sqrt{r^2 - x^2})^2\,dx = \frac{4}{3}\pi r^3$